

$$\left\{ \frac{n}{\sqrt{1+4n^2}} \right\}$$

$$\lim_{n \rightarrow \infty} \frac{n}{\sqrt{1+4n^2}} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{\frac{1}{n^2} + 4}} \quad \textcircled{2}$$

$$= \frac{1}{\sqrt{0+4}}$$

$$= \frac{1}{2} \quad \textcircled{1}$$

$$\sum_{n=1}^{\infty} \frac{2 + 2^{2n}}{5^n}$$

$$= \sum_{n=1}^{\infty} 2 \cdot \left(\frac{1}{5}\right)^n + \sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n$$

(BOTH GEOMETRIC SERIES)  
 $r = \frac{1}{5}$  AND  $r = \frac{4}{5}$

$$= \frac{\frac{2}{5}}{1 - \frac{1}{5}} + \frac{\frac{4}{5}}{1 - \frac{4}{5}}$$

$$= \frac{1}{2} + 4$$

$$= \frac{9}{2}$$

$$\sum_{n=1}^{\infty} 3^{\frac{1-n}{n}}$$

$$\lim_{n \rightarrow \infty} \frac{1-n}{n} = -1 \quad (1)$$

(1)

$$\text{so } \lim_{n \rightarrow \infty} 3^{\frac{1-n}{n}} = 3^{-1} = \frac{1}{3} \neq 0$$

By DIVERGENCE TEST

$$\sum_{n=1}^{\infty} 3^{\frac{1-n}{n}} \text{ DIVERGES}$$

(1)

$$\left\{ \frac{\sin n}{e^n} \right\}$$

$$-1 \leq \sin n \leq 1 \quad \text{for } n \in \mathbb{Z}^+$$

$$\textcircled{1} \quad \left[ -\frac{1}{e^n} \leq \frac{\sin n}{e^n} \leq \frac{1}{e^n} \right]$$

$$\textcircled{1} \quad \left[ \lim_{n \rightarrow \infty} -\frac{1}{e^n} = \lim_{n \rightarrow \infty} \frac{1}{e^n} = 0 \right]$$

BY SQUEEZE THEOREM

$$\textcircled{1} \quad \left[ \lim_{n \rightarrow \infty} \frac{\sin n}{e^n} = 0 \right]$$

Find all values of  $x$  for which  $\sum_{n=0}^{\infty} 2^{n+1}(3-x)^n$  is convergent. You do NOT need to find the sum.

SCORE: \_\_\_\_ / 4 PTS

$$= 2 + 2^2(3-x) + 2^3(3-x)^2 + \dots$$

GEOMETRIC SERIES

$$r = 2(3-x)$$

$$|2(3-x)| < 1 \quad (2)$$

$$-1 < 2(3-x) < 1$$

$$-\frac{1}{2} < 3-x < \frac{1}{2}$$

$$-\frac{7}{2} < -x < -\frac{5}{2}$$

$$\frac{5}{2} < x < \frac{7}{2} \quad (2)$$

Determine if each sequence below is increasing, decreasing or neither.

SCORE: \_\_\_\_ / 5 PTS

Justify each answer using proper mathematical reasoning and/or algebra.

Your solutions must NOT use derivatives.

[a]  $\left\{ \frac{3n-5}{4n-3} \right\}$

$$\begin{aligned} a_{n+1} - a_n &= \frac{3(n+1)-5}{4(n+1)-3} - \frac{3n-5}{4n-3} \\ &= \frac{3n-2}{4n+1} - \frac{3n-5}{4n-3} \quad \textcircled{1} \\ &= \frac{11}{(4n+1)(4n-3)} \quad \textcircled{1} \end{aligned}$$

$$\left. \begin{aligned} a_{n+1} - a_n &> 0 \text{ FOR } n \in \mathbb{Z}^+ \\ a_{n+1} &> a_n \text{ FOR } n \in \mathbb{Z}^+ \end{aligned} \right] \textcircled{1} \text{ EITHER STATEMENT IS OK}$$

INCREASING  $\textcircled{1}$

[b]  $\left\{ \frac{5n-11}{2n-5} \right\} = \{2, 1, 4, \dots\} \quad \textcircled{\frac{1}{2}}$   
NEITHER  $\textcircled{\frac{1}{2}}$

Consider the following statements.

SCORE: \_\_\_\_ / 3 PTS

- (i) If  $\{a_n\}$  has limit 0, then  $\sum_{n=1}^{\infty} a_n$  is convergent
- (ii) If  $\sum_{n=1}^{\infty} a_n$  is divergent, then  $\{a_n\}$  diverges
- (iii) If  $\{a_n\}$  is bounded, then  $\{a_n\}$  converges

Which of the statements above are true ? Circle the correct answer below.

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[a] none are true

[b] only (i) is true

[c] only (ii) is true

[d] only (iii) is true

[e] only (i) and (ii) are true

[f] only (i) and (iii) are true

[g] only (ii) and (iii) are true

[h] all are true